Well-being and Demographic Dynamics

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ABSTRACT

This paper presents a socio-demographic model defined by age and sex.

The model is a von Foerster-McKendrick model for the dynamics of population per sex and age of a general human population.

The fertility and deaths rates are defined by age and they depend on the well-being variable.

The well-being variable is defined by UN: HDI-Hybrid.

We present the validation of the stochastic formulation model from the case of Spain in the period 2003-2008.

INDEX

- Introduction
- The Model
  - Previous Model
  - Last Model
  - Rates
    - Fertility
    - Death
    - Migration
- Validation
  - Stochastic
- Conclusion

INTRODUCTION

This paper presents a socio-demographic model defined by sex and structured by age.

The **objective** is to study the evolution of human population. The population variables are defined by two sexes and they are structured by age.

We contemplate the relationship between a well-being variable and the population variables. The **Hybrid Human Development Index** is the considered well-bing variable. It was defined by the UN in his Human Development Report (UNDP, 2010).

INTRODUCTION II

There are other papers in which they work with socio-demographic models. These models are structured by sex and they use some well-being variables

Caselles et al. (2008) present a socio-demographic temporal model. The model does not difference sex, but fertility and mortality rates depend on *HDI*.

Sanz et al. (in press) present a socio-demographic temporal model per sex. In this case, the demographic rates depend on *GEM*.

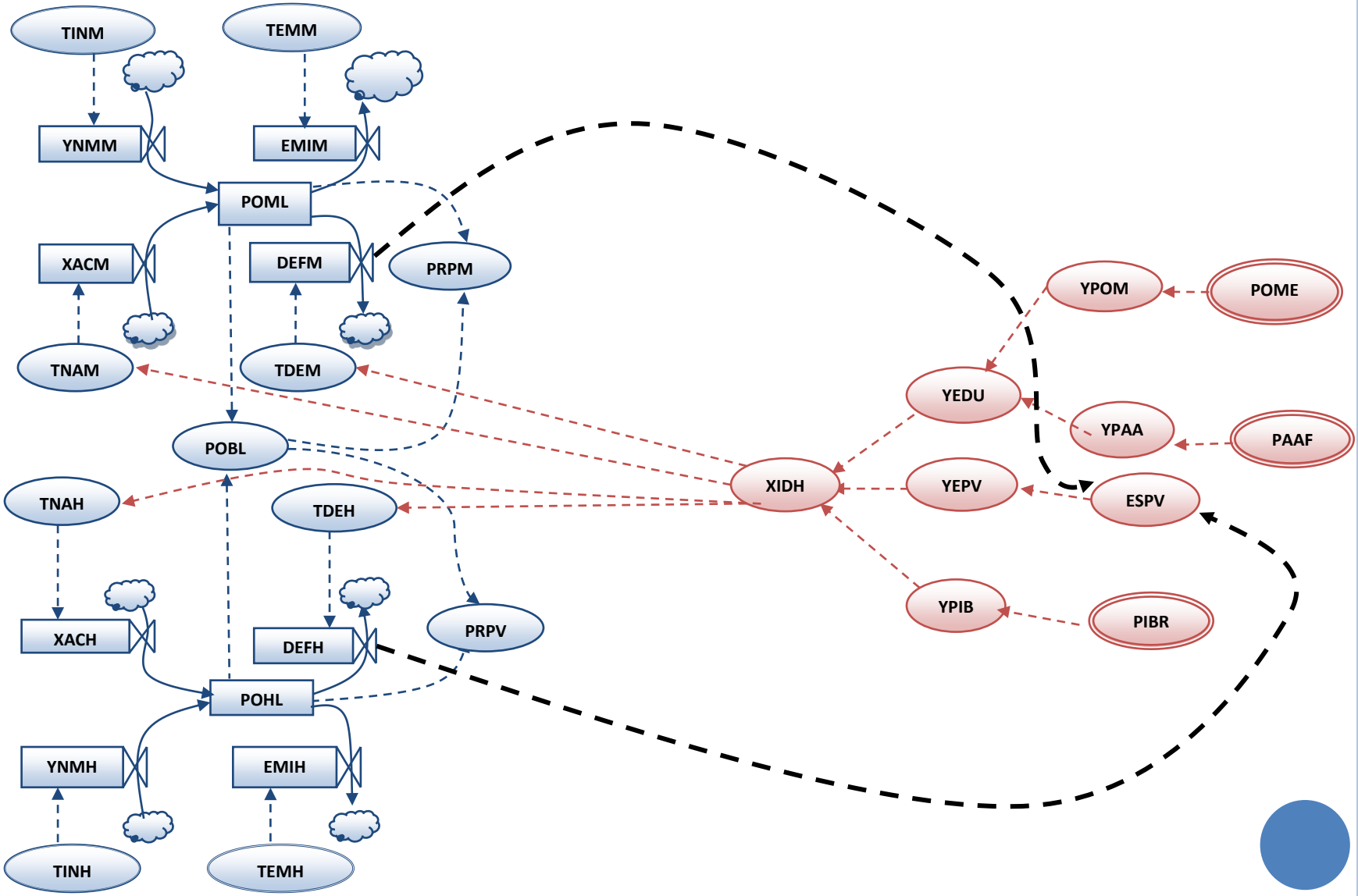
Finally, Sanz et al. (2011, 2012) present a socio-demographic temporal model per sex. In this case, the demographic rates depend on the interaction between three different indices: *GEM*, *GDI* and *HDI*.

INTRODUCTION III

The model presented here contains one well-being variable defined by UN in the Human Development Report of 2010 (UNDP, 2010), the *HDI-Hybrid*. This index is a redefinition of the previous Human Development Index, present in the past reports (UNDP, 1998-2009). It is still an aggregate measure of progress in three basic dimensions: health, education and income.

This new index is named *HDI-Hybrid*, **(only changes the calculation process respect to the old index: geometric mean substitutes arithmetic mean of the three component indicators)**.

THE MODEL. FORRESTER'S DIAGRAM



THE PREVIOUS MODEL.

The initial structure of the model is the same as presented in Mico et al. (2008), which presents von Foerster-McKendrick model for the population dynamics per sex and age of a general human population

Death Rate

Population Density

$$\frac{\partial w_i(t,x)}{\partial t} + c \cdot \frac{\partial w_i(t,x)}{\partial x} = -b_i(t,x) \cdot w_i(t,x) + m_i(t,x),$$

Migration Rate

$$w_i(t,0) = \int_0^{\infty} a_i(t,x) \cdot w_2(t,x) \cdot dx - b_i(t,0) \cdot w_i(t,0) + m_i(t,0),$$

Fertility Rate

$$w_i(t_0,x) = u_i(x)$$

Initial Population Density

THE LAST MODEL.

We try to get the relation between *HDI-Hybrid* and fertility and death rates

$$a_i(t,x) = A_i(hdi(t),x) \qquad b_i(t,x) = B_i(hdi(t),x)$$

$$\frac{\partial w_i(t,x)}{\partial t} + c\cdot\frac{\partial w_i(t,x)}{\partial x} = -B_i(hdi(t),x)\cdot w_i(t,x) + (f_i(x) - g_i(x))\cdot w_i(t,x),$$

$$w_i(t,0) = \int_0^\infty A_i(hdi(t),x)\cdot w_2(t,x)\cdot dx - B_i(hdi(t),0)\cdot w_i(t,0) + (f_i(0) - g_i(0))\cdot w_i(t,0),$$

$$w_i(t_0,x) = u_i(x)$$

THE RATES. FERTILITY RATES

The data used correspond to the initial year: 2002. These data are the respective births of males and females per women of childbearing age (between 14 and 50, present in the statistical database).
Respect to the independent variable, it is the quotient between the cohort number and the well-being index. The fitted function is a linear combination of two positive Gaussian functions.

$$A_i(hdi(t),x)=\alpha_{i0}+\alpha_{i1}e^{-\frac{\left(\frac{x}{hdi(t)}-\mu_{i1}\right)^2}{2\cdot\beta_{i1}}}+\alpha_{i2}e^{-\frac{\left(\frac{x}{hdi(t)}-\mu_{i2}\right)^2}{2\cdot\beta_{i2}}}$$

THE RATES. DEATH RATES

The data used correspond to the initial year: 2002. These data are the deaths of males and females per unit in the respective cohort.
The fitted function is considered as a piecewise function.

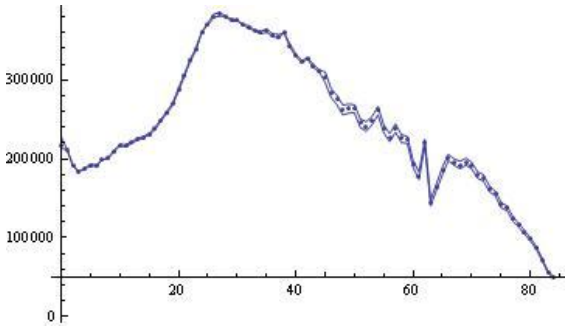
$$B_i(calidad(t),x)=\begin{cases}\alpha_{i0}+\alpha_{i1}e^{-\frac{x}{\frac{calidad(t)}{\beta_{i1}}}}+\sum_{j=2}^{4}\alpha_{ij}e^{-\frac{\left(\frac{x}{calidad(t)}-\mu_{ij}\right)^2}{2\cdot\beta_{ij}}} & 0<x<45\\ \alpha_{i5}+\alpha_{i6}e^{-\frac{x}{\frac{calidad(t)}{\beta_{i6}}-\mu_{i6}}}+\sum_{j=7}^{8}\alpha_{ij}e^{-\frac{\left(\frac{x}{calidad(t)}-\mu_{ij}\right)^2}{2\cdot\beta_{ij}}} & 46<x<100\end{cases}$$

THE RATES. MIGRATION RATES

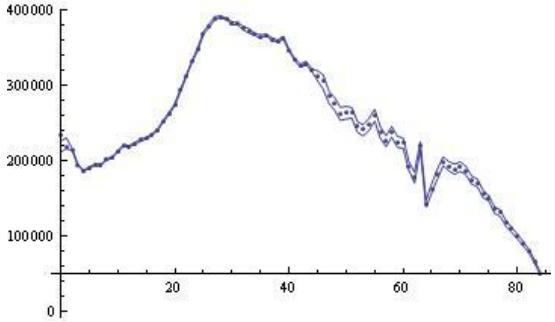
The data used correspond to the initial year: 2002. These data are the migration balance of males and females per unit in the respective cohort.

The independent variable is the corresponding cohort number. In this case well-being variables are not introduced in formulas. In the future, we should consider quality of life in the host country and quality of life in the departure country.

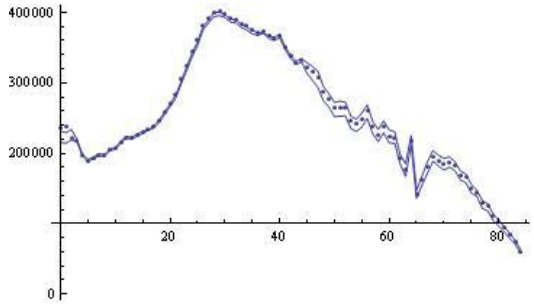
VALIDATION. MALE POPULATION. STOCHASTIC



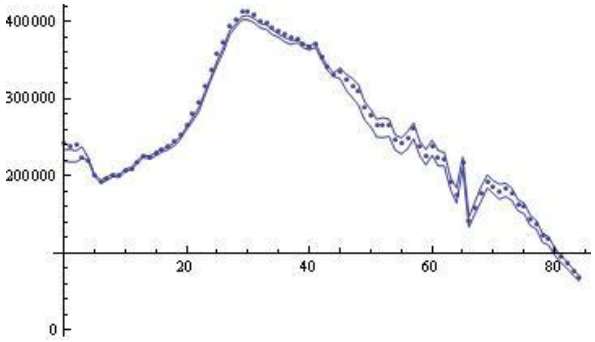
Male Population 2003.



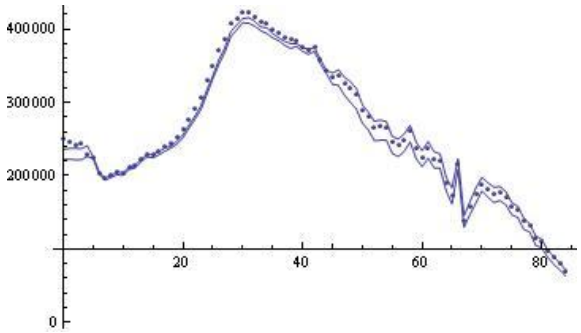
Male Population 2004.



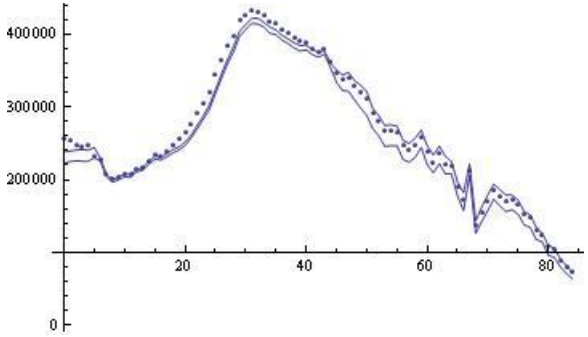
Male Population 2005.



Male Population 2006.

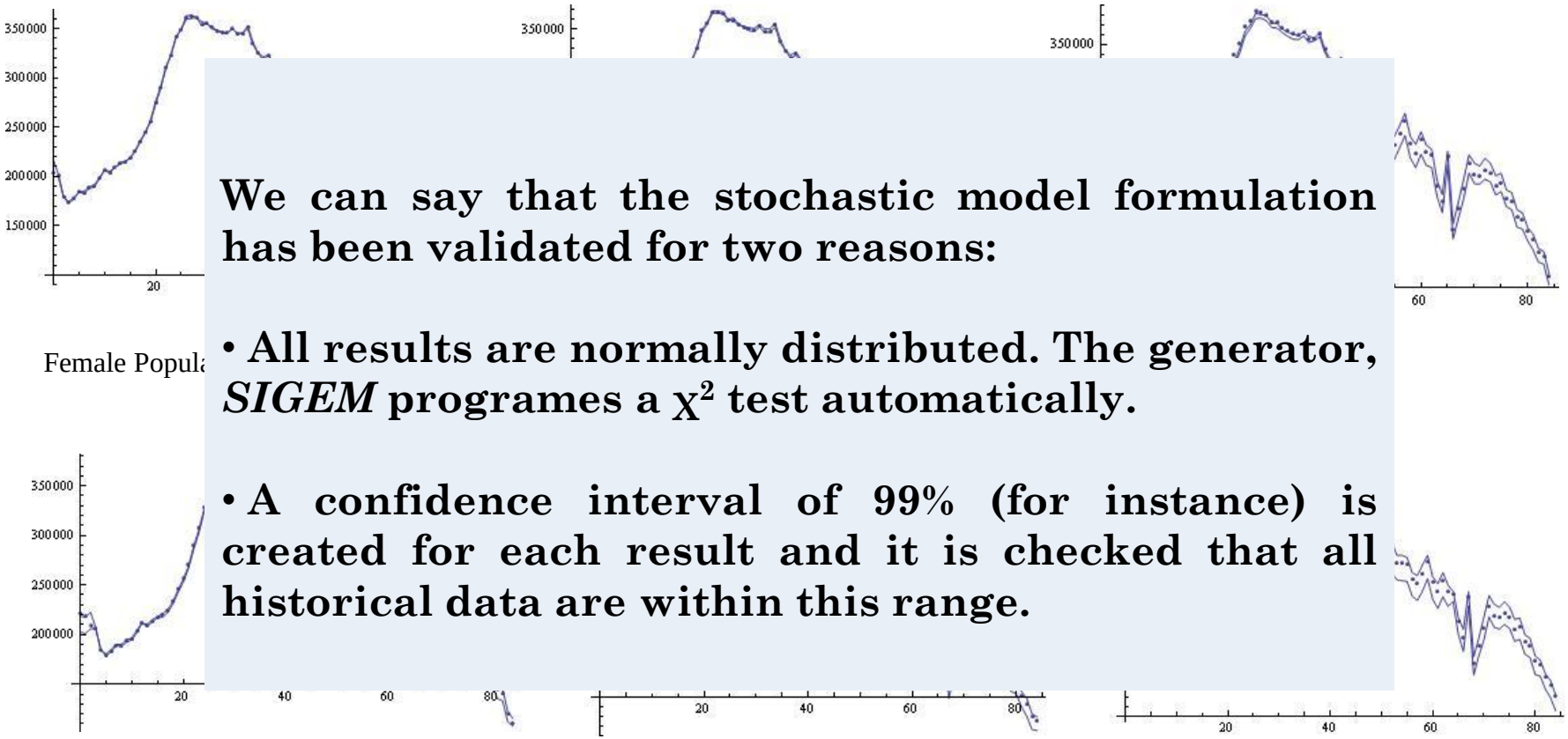


Male Population 2007.



Male Population 2008.

VALIDATION. FEMALE POPULATION. STOCHASTIC



We can say that the stochastic model formulation has been validated for two reasons:

- **All results are normally distributed. The generator, *SIGEM* programes a χ^2 test automatically.**
- **A confidence interval of 99% (for instance) is created for each result and it is checked that all historical data are within this range.**

Female Population 2006. Female Population 2007. Female Population 2008.

CONCLUSION I

PRESENT

A mathematical model is presented to **study the sex-and-age structured human population dynamics in relation to the *HDI-Hybrid* United Nations wellbeing index.**

CONCLUSION II

PRESENT

A model is presented to undertake a more detailed and complete study: the stochastic model.

All demographic variables: fertility, death, emigration and immigration rates are defined per sex and structured by age.

The corresponding fitted functions obtained high determination coefficient values, R^2.

Then, **the stochastic model formulation is validated for the case of Spain in the 2003-2008 period**.

CONCLUSION III

FUTURE

One future possibility is to **obtain a model in which the demographic rates are related with all well-being variables** (variables defined by the *UN*).

CONCLUSION IV FUTURE

Other future research works could consist in attempts to **validate the here presented model in other countries**.

But in this case these countries must try to obtain **good statistical data bases**.

CONCLUSION V

FUTURE

Finally, another possible ambitious work is **to find a model that considers several countries or regions, in which the migration rates could be fitted in terms of the well-being variables corresponding to the involved countries**.

THANKS

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